Artur Avila **Thursday, July 17, 10:30–11:15, Room B2**
(Laboratoire de Probabilités, Université Paris VI, France)
Dynamics of quasiperiodic cocycles and the spectrum of the almost Mathieu operator

Abstract: The almost Mathieu operator $H = H_{\lambda,\alpha,\theta} : \ell^2(\mathbb{Z}) \to \ell^2(\mathbb{Z})$ is given by

$$(Hu)_n = u_{n+1} + u_{n-1} + 2\lambda \cos 2\pi(\theta + n\alpha)u_n,$$

where λ (the coupling), α (the frequency) and θ (the phase) are parameters. Originally introduced and studied in the physics litterature, it turned out to also give rise to a rich mathematical theory, where algebra, analysis and dynamical systems interact. Among some key distinguishing features, we point out the presence of a remarkable symmetry (Aubry duality) between large and small couplings, and a sharp phase transition at the self-dual point $\lambda = 1$.

We stress that there is also qualitative dependence with respect to the frequency and the phase (related to their Diophantine properties), and which makes it particularly tough to achieve a description of the whole parameter space. Neverthless key natural questions (regarding the topology and measure of the spectrum, but also the nature of the spectral measures) have recently been fully addressed.

We will discuss these and related questions which focused the developments since 1980, emphasizing the connection with the dynamics of quasiperiodic cocycles which has played a fundamental role in the latest contributions.